Now that we have discussed both permutations and combinations independently, it's time to look at questions that involve both. Mind you, these questions are not difficult -– they just involve both concepts. The first one is a circular arrangement question with a tiny twist. The second one requires us to make some cases. It takes a fair bit of patience to work out one case at a time and I doubt that GMAT will give you such a question since it is a little bit of a bore. (Actual GMAT questions have more entertainment value for the test maker and the test taker. They make you think and are FUN to solve) That said, it is a great question to bind together everything that we have learned till now and strengthen your understanding. Let's start.

Question 1: Seven women and four men have to sit around a circular table so that no two men are together. In how many different ways can this be done?

Solution: Try and think about it for a while. We did a very similar question while working on circular arrangements. In that question, number of women and number of men were equal so we just had to place them in alternate positions. Here, we have fewer men. What do we do now?

Two men cannot sit together but some women will sit together since there aren't enough men. So, let's make the 7 women sit around the round table in (7-1)! = 6! ways (We covered the (n-1)! concept in the post on circular arrangements)

Now, how many places do we have for the men? A man can sit between any two women sitting next to each other. How many such pairs of women are there? Since there are 7 women, we have 7 such pairs and hence 7 possible spaces for men. There are two different approaches you can take from here:

Approach 1:

We have 4 men but 7 possible spaces for them. For the first man, we can select a space in 7 ways. For the second man, we can select a space in 6 ways. For the third one, in 5 ways and for the fourth one in 4 ways. So we can arrange the men in 7*6*5*4 ways. This is just our basic counting principle in action.

Approach 2:

Some people like to split up the task into two steps – make the selection, then arrange. Out of 7 spaces, we need to select any 4 for the 4 men. How do you select 4 out of 7? Using basic counting principle and un-arranging concept, we can do it in 7*6*5*4/4! ways (or we can use the formula $_7C_4$). We have selected 4 spaces so now we just want to arrange the 4 women in the 4 spaces. We can do this in 4! ways.

It doesn't matter which approach you use. The first one uses just the basic counting principle. The second one is used more often by people who are very comfortable with the combinations formula.

The total number of arrangements we get = 6! * 7*6*5*4 or we can write this as 6!*7!/3! to make it a little compact.

Let's look at the second question now.

Question 2: How many words of 4 letters can be formed from the word "INFINITY"? (They may or may not be actual words in the English language.)

The word INFINITY has 5 distinct letters – I, N, F, T, Y

Repetitions – I, I, I, N, N

The question doesn't say that all letters of the words have to be distinct. So, you can make a word using all three Is and another letter or two Ns and two Is etc. So you cannot just select any four letters and arrange them. The number of arrangements will vary depending on whether the letters are all distinct or have some repetitions. Let's look at all possible cases:

Case 1: All letters are distinct (Form: abcd)

From the 5 distinct letters, we can select any 4 (or drop any 1 letter) in 5 ways (you can also use $_5C_4$ or 5*4*3*2/4! to arrive at the figure of 5)

We can arrange these 4 selected letters in 4! ways.

Number of ways in which you can make a 4 letter word with all distinct letters = 5*4! = 120 ways

Case 2: Two letters same, others different (Form: aabc)

Only I and N are repeated so we have to select one of them and we have to select 2 of the other 4 (F, T, Y and whatever is not selected out of N and I) letters.

Select one of N and I in 2 ways. Then to choose 2 other letters, pick two from the other 4 letters in 4*3/2 ways (or 4C2) = 6 ways.

Now we have 3 letters and one of them is repeated so in all we have 4 letters. We can arrange 4 letters (with a repetition) in 4!/2! ways (we divide by 2! because one letter is repeated).
Number of ways in which you can make a 4 letter word with one repetition = 2 * 6 * 4!/2! = 144 ways
Case 3: 2 letters, both repeated (Form: aabb)
We have only two letters that are repeated, N and I. We will need to select both of them so the selection can be done in only 1 way.
Since both the letters are repeated, the 4 letter word can be formed in 4!/(2!*2!) = 6 ways
Case 4: 3 letters same, fourth different (Form: aaab)
Only I appears 3 times so it must be selected. We have to select one letter from the other four. We can choose the fourth letter in 4 ways.
Since I is repeated 3 times, the four letters can be arranged in 4!/3! = 4 ways.
Number of ways in which you can make a 4 letter word 4 * 4 = 16 ways
All four letters cannot be the same since no letter appears four times.
Total number of 4 letter words that can be formed using the letters of the word ‘INFINITY’ are 120 + 144 + 6 + 16 = 286 words
The solution is long but very methodical. If you go one step at a time, it is not complicated at all. I will see you next week with some tricky questions. Till then, keep practicing!